NUCLEAR STRUCTURE ASPECTS OF THE NEUTRINOLESS DOUBLE BETA DECAY

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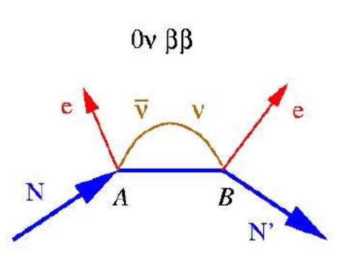

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[1] E. Caurier and F. Nowacki, Strasbourg-Madrid Shell-Model collaboration

Double beta decays

Some, otherwise nearly stable, nuclei decay emitting two electrons and two neutrinos ($2\nu\ \beta\beta$) by a second order process mediated by the weak interaction, that has been experimentally measured in several favorable cases. The decay probability contains a phase space factor and the square of a nuclear matrix element

$$[T_{1/2}^{2\nu}]^{-1} = G_{2\nu}|M_{GT}^{2\nu}|^2$$

If the neutrinos are massive Majorana particles, the double beta decay can take place without emission of neutrinos ($0\nu\ \beta\beta$). In this case the transition is mediated by terms that go beyond the standard model. The decay probability contains a phase space factor, the effective electron neutrino mass (a linear combination of the mass eigenstates whose coefficients are elements of the mixing matrix) and the nuclear matrix element

$$[T_{1/2}^{(0\nu)}(0^+ -> 0^+]^{-1} = G_{0\nu}\left(M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)}\right)^2 \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2$$

Ingredients of the calculations of double beta decays

2ν

The matrix element is:

$$M_{GT}(2\nu) = \sum_i \frac{\langle GD|\vec{\sigma}\cdot\vec{\tau}|i\rangle\langle i|\vec{\sigma}\cdot\vec{\tau}|F\rangle}{E_i}$$

- We need the wave function of the ground state of the father nucleus (F) and of the ground state and occasionally a few excited states of the grand daughter (GD)
- We need the wave function of all the 1^+ states (I) in the odd-odd daughter nucleus
- The operator is well under control
- The spin-isospin operators in the nuclear medium are quenched by a factor $1/g_A$

Ingredients of the calculations of double beta decays

0ν

To compute exactly the 0ν matrix element, we would also need the wave function of the ground state of the father nucleus (F) and of the ground state grand daughter (GD), but in addition we need all the J^{π} excited states in the odd-odd daughter nucleus

Most conveniently, the matrix elements in the closure approximation, that is good to better than 90% due to the high neutrino momentum (≈100 MeV), look like:

$$M_{GT}(0\nu) = \frac{\langle GD|h(|\vec{r_1} - \vec{r_2}|)(\vec{\sigma}_1 \cdot \vec{\tau}_1)(\vec{\sigma}_2 \cdot \vec{\tau}_2)|F\rangle}{\langle E_\nu + E_i\rangle}$$

where $h(|\vec{r_1} - \vec{r_2}|)$ are the neutrino potentials (≈1/r)
In this approximation no knowledge of the intermediate nucleus is directly implied

Ingredients of the calculations of double beta decays

0ν

However, some of the elements entering in the calculation, beyond the nuclear wave functions themselves, are still under debate

- The transition operators are usually obtained from the Hamiltonian of Doi et al. However, according to recent claims (Simkovic et al.), additional terms originating in the coupling to the virtual pions, should give non-negligible contributions
- The need or not of quenched spin-isospin operators is not settled (but I believe this is not an issue)
- The short range correlations, have to be taken into account in the calculation of the two body matrix elements of the 0ν two-body transition operators, but, is the Jastrow ansatz enough?

How does the GT 0ν operator act?

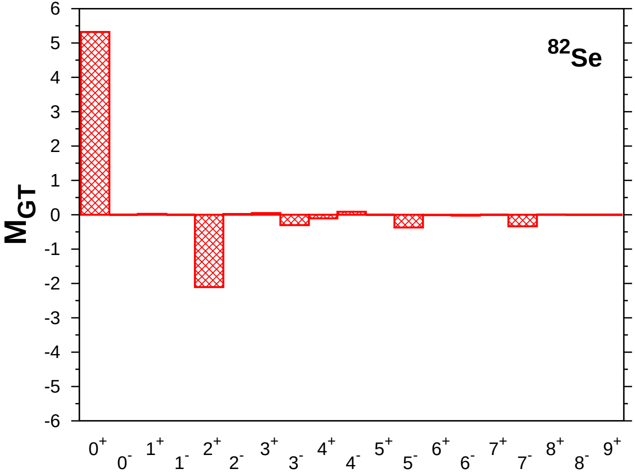
The two body transition operators can be written generically as:

$$M_{GT}(0\nu) = \sum_J \langle GD| \sum_{i,j,k,l} M^J_{i,j,k,l} \; ((a^\dagger_i a^\dagger_j)^J \; (a_k a_l)^J)^0 |F\rangle$$

In words, what the operator does is; first, to annihilate two neutrons in the parent nucleus, next, to create two protons, and finally to overlap the resulting object with the grand daughter ground state.

The contributions to the 0ν matrix element as a function of the **J** of the of the decaying pair:

$^{82}Se \rightarrow {}^{82}Kr$



The multipole structure of the 0ν matrix element

The two body transition operators can also be be written as:

$$M_{GT}(0\nu) = \sum_{\lambda} \langle GD | \sum_{i,j,k,l} \Omega^{\lambda}_{i,j,k,l} \left((a^{\dagger}_i a_k)^{\lambda} \, (a^{\dagger}_j a_l)^{\lambda} \right)^0 | F \rangle$$

The "crisis" of the calculations of the 0ν, $\beta\beta$ nuclear matrix elements

The QRPA "explosion"

- g_{pp}, the miraculous factor
- Does a good 2ν m.e. guarantee a good 0ν m.e.?
- To quench or not to quench ...
- Short range correlations ...
- Higher order terms of the nucleon current, e.g. induced pseudoscalar term)

The quest for better wave functions

Quality indicators

- Good spectroscopy for parent, daughter and grand-daughter, even better if its extend to a full mass region
- GT-strengths and strength functions, 2ν matrix elements, etc.

Large scale shell model (LSSM) *vs* QRPA calculations

- Interaction
  - LSSM: Monopole corrected G-matrices
  - QRPA: Realistic or schematic interactions tuned with the g_{ph} and g_{pp} strengths
- Valence space
  - LSSM: "Small", but all the possible ways of distributing the valence particles among the valence orbits are taken into account.
  - QRPA: "Large", but only 1p-1h and 2p-2h excitations from the normal filling are considered (and not all of them)

Large scale shell model (LSSM) *vs* QRPA calculations

- Pairing
  - LSSM: It is treated exactly in the valence space. Proton and neutron numbers are exactly conserved. Proton-proton, neutron-neutron, and proton-neutron (isovector and isoscalar) pairing is included
  - QRPA: Only proton-proton, and neutron-neutron pairing is considered. It is treated in the BCS approximation. Proton and neutron numbers are not exactly conserved
- Deformation
  - LSSM: Described properly in the laboratory frame. Angular momentum conservation preserved
  - QRPA: Not incorporated

The three pillars of the shell model

The Effective Interaction
The Valence Space
The Algorithms and the Codes

E. Caurier, G. Martínez-Pinedo, F. Nowacki,
A. Poves and A. P. Zuker.
"The Shell Model as a Unified View of Nuclear Structure"
Reviews of Modern Physics, 77 (2005) 427-488

The Effective Interaction: Key aspects

The evolution of the Spherical mean field in the valence spaces. Usually the realistic interactions require some monopole adjustments

The multipole hamiltonian (pairing, quadrupole, etc.) extracted from the realistic interactions seems to be universal and correct

The Valence Space(s)

Miscellanea of computationally accessible valence spaces:
(note: in a major HO shell of principal quantum number **p** the orbit j=p+1/2 is called *intruder* and the remaining ones are denoted by r_p)

- Classical $0\hbar\omega$ valence spaces are the *p*, *sd* and *pf* shells
- r_2-*pf*: intruders around N and/or Z=20
- *sd*-protons, *pf*-neutrons: very neutron rich Mg, Al, Si, P, S, Cl, Ar, and K
- r_3-$g9/2(d5/2)$: ^{76}Ge, ^{82}Se, and the region around ^{80}Zr
- r_3-$g9/2(d5/2)$ for the neutrons and *pf* for protons: neutron rich Cr, Fe, Ni, Zn
- r_4-$h_{11/2}$ for neutrons and $p_{1/2} - g_{9/2}$-r_4 for protons: ^{96}Zr, ^{100}Mo, ^{110}Pd, ^{116}Cd
- r_4-$h_{11/2}$ for neutrons and protons: ^{124}Sn, $^{128-130}$Te, ^{136}Xe

The Algorithms and the Codes

Algorithms include Direct Diagonalisation, Lanczos, Monte Carlo Shell Model, Quantum Monte Carlo Diagonalization, Projected Shell-Model, DMRG and different extrapolation ansatzs

The Strasbourg-Madrid codes can deal with problems involving basis of 10^{10} Slater determinants, using relatively modest computational resources

Update of the 0ν results

In the valence spaces r_3-$g_{9/2}$ (^{76}Ge, ^{82}Se) and r_4-$h_{11/2}$ (^{124}Sn, $^{128-130}$Te, ^{136}Xe) we have obtained high quality effective interactions by carrying out multi-parametrical fits whose starting point is given by realistic G-matrices. In the valence space proposed for ^{96}Zr, ^{100}Mo, ^{110}Pd and ^{116}Cd, the results are still to come or subject to further improvement

	m_ν for $T_{\frac{1}{2}} = 10^{25}$ y.	$M_{0\nu}^{GT}$	$1-\chi_F$
^{48}Ca	0.85	0.67	1.14
^{76}Ge	0.90	2.35	1.10
^{82}Se	0.42	2.26	1.10
(^{110}Pd)	0.67	2.21	1.15
(^{116}Cd)	0.27	2.49	1.18
^{124}Sn	0.45	2.11	1.13
^{128}Te	1.92	2.36	1.13
^{130}Te	0.35	2.13	1.13
^{136}Xe	0.41	1.77	1.13

LSSM *vs* QRPA 0ν results

$M_{0\nu}$	LSSM	QRPA(1)	1+(hoc)	QRPA(hoc)(2)
^{76}Ge	2.58	3.60	2.80	2.40
^{82}Se	2.49	3.40	2.64	2.12
(^{116}Cd)	2.94	2.58	2.05	1.43
^{128}Te	2.67	2.96	2.17	1.60
^{130}Te	2.41	2.50	1.80	1.47
^{136}Xe	2.00	1.02	0.66	0.98

(hoc), including higher order corrections, their need is still under debate
(1) renormalized QRPA with the standard value of the strength of the particle-particle interaction. Simkovic, Pantis, Vergados, Faessler (1999)
(2) renormalized QRPA with the value of the strength of the particle-particle interaction adjusted to the 2ν lifetimes on a nucleus by nucleus basis (also under debate) Rodin, Faessler, Simkovic, Vogel (2006)

Dependence on the effective interaction

The results depend only weakly on the effective interactions provided they are compatible with the spectroscopy of the region.
For the lower *pf* shell we have three interactions that work properly, KB3, FPD6 and GXPF1. Their predictions for the 2ν and the neutrinoless modes are quite close to each other

	KB3	FPD6	GXPF1
$M_{GT}(2\nu)$	0.083	0.104	0.107
$M_{GT}(0\nu)$	0.667	0.726	0.621

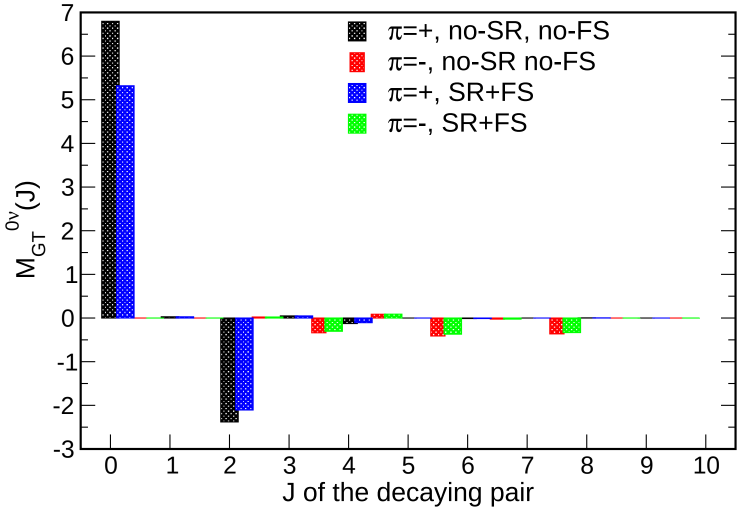
Similarly, in the *r3g* and *r4h* spaces, the variations among the predictions of spectroscopically tested interactions is small (10-20%)

Finite size and short range corrections

$M_{GT}(0\nu)$	bare	+SR	+FS	+SR+FS
$^{48}Ca \rightarrow ^{48}Ti$	1.211	0.735	0.930	0.668
$^{48}Ti \rightarrow ^{48}Cr$	1.990	1.408	1.631	1.298
$^{76}Ge \rightarrow ^{76}Se$	3.515	2.241	2.643	2.350
$^{82}Se \rightarrow ^{82}Kr$	3.329	2.103	2.478	2.253

Finite size and short range corrections

The effect of the SR and FS corrections proceeds mainly through the reduction of the $\mathbf{J}^{\pi}\mathbf{=0}^{+}$ pair contribution



The influence of the spin-orbit partner

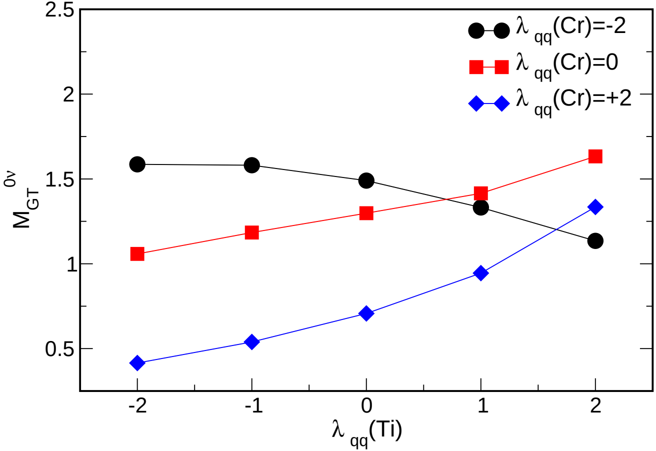
We can increase artificially the excitation energy of the spin-orbit partner of the intruder orbit. Surprisingly enough, this affects very little the values of the matrix elements, particularly in the neutrinoless case. Even removing the spin-orbit partner completely produces minor changes

	^{48}Ca $\rightarrow$ ^{48}Ti	^{48}Ti $\rightarrow$ ^{48}Cr	
$M_{GT}(2\nu)$	0.083	0.213	SO-partner-in
$M_{GT}(2\nu)$	0.049	0.274	SO-partner-out
$M_{GT}(0\nu)$	0.667	1.298	SO-partner-in
$M_{GT}(0\nu)$	0.518	1.386	SO-partner-out

The influence of deformation

Changing adequately the effective interaction we can increase or decrease the deformation of parent, grand-daughter or both, and so gauge its effect on the decays. We have artificially changed the deformation of ^{48}Ti and ^{48}Cr adding an extra $\lambda Q \cdot Q$ term to the effective interaction. A mismatch of deformation can reduce the $\beta\beta$ matrix elements by factors 2-3. This exercise shows that the effect of deformation is very important and cannot be overlooked. Similar results are obtained for heavier nuclei

The influence of deformation

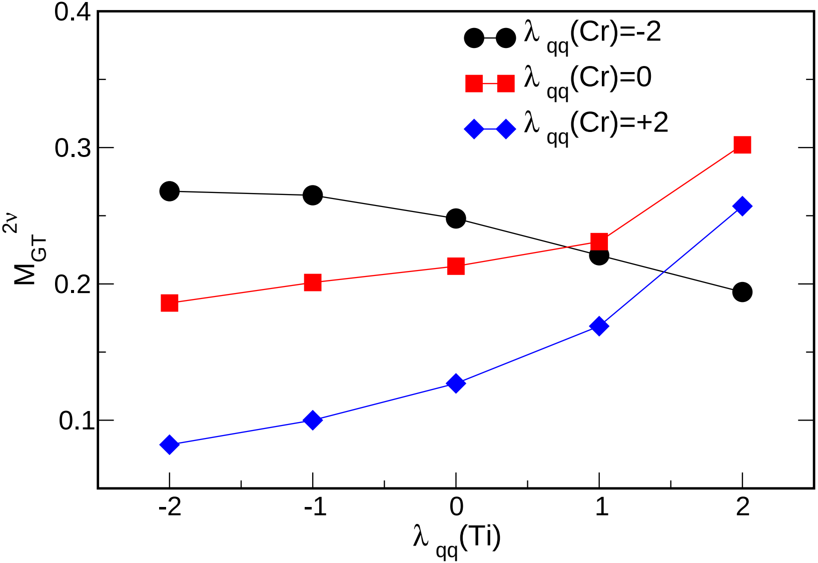


black circle to the left, spherical-spherical

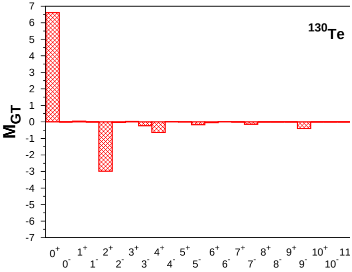
red square to the right, equally deformed Ti and Cr

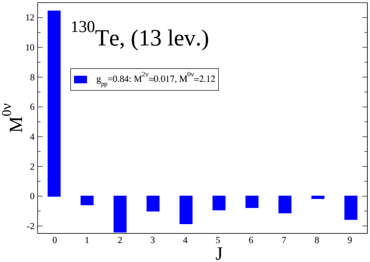
blue diamond to the left, spherical Titanium, very deformed Chromium

The influence of deformation; the 2ν case



The contributions to the 0ν matrix element as a function of the **J** of the of the decaying pair : A=130





LSSM calculations in QRPA-like valence spaces: ^{82}Se

The SM valence space for the ^{76}Ge and ^{82}Se decays has been traditionally:

$$1p_{\frac{3}{2}},\ 0f_{\frac{5}{2}},\ 1p_{\frac{1}{2}},\ 0g_{\frac{9}{2}}$$

Although only recently full calculations in this space have been possible.

In the QRPA, it is rather:

$$0f_{\frac{7}{2}},\ 1p_{\frac{3}{2}},\ 0f_{\frac{5}{2}},\ 1p_{\frac{1}{2}},\ 0g_{\frac{9}{2}},\ 1d_{\frac{5}{2}},\ 0g_{\frac{7}{2}},\ 2s_{\frac{1}{2}},\ 1d_{\frac{3}{2}}$$

LSSM calculations in QRPA-like valence spaces: ^{82}Se

As a first step toward a more complete benchmarking, we have evaluated the influence of the 2p-2h jumps from the $1f_{7/2}$ orbit – ^{56}Ni core excitations– in our results for the ^{82}Se decay. Similar calculations for the ^{76}Ge decay are under way

The calculation in the full r3g space plus 2p-2h proton excitations from the 0f7/2 orbit gives a 20% increase of $M^{0\nu}$, but probably we overestimate the amount of core excitations. Our f7/2 proton occupancies, 7.71 and 7.69 in ^{82}Se and ^{82}Kr are smaller than the BCS occupancies of Rodin et al. 7.84 and 7.84. Therefore the above 20% must be taken as an upper bound

The 2ν matrix element remains nearly constant, even if the total Gamow-Teller strengths, (GT+) and (GT-), increase from 0.15 to 0.34 and from 20.5 to 26.9

LSSM calculations in QRPA-like valence spaces: ^{136}Xe

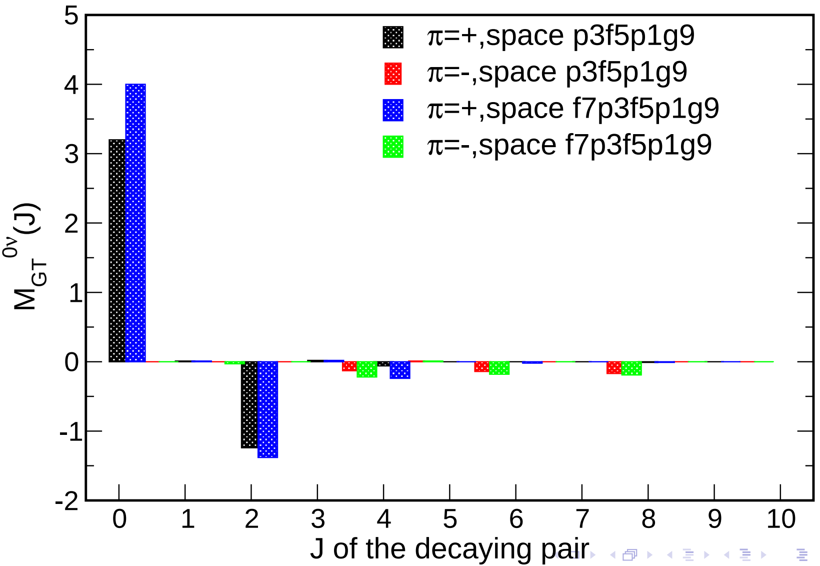
We have also computed the ^{136}Xe decay in the r4h space including 2p-2h excitations from the 0g9/2 proton orbit and the matrix element increases less than 10%

In another set of calculations, we have included 2p2h neutron excitations toward the 0h9/2 and 1f/7/2 orbits. The occupancies that we obtain are relatively large (0.25 neutrons in each orbit) and the effect is to increase the matrix element by 15%. It is interesting to note that the increase with the two orbits simultaneously active is equivalent to that obtained including one or another orbit separately. Therefore there is no pile-up of the contributions of the small components of the wave function.

THE LSSM results seem to be robust against the inclusion of small components of the wave function

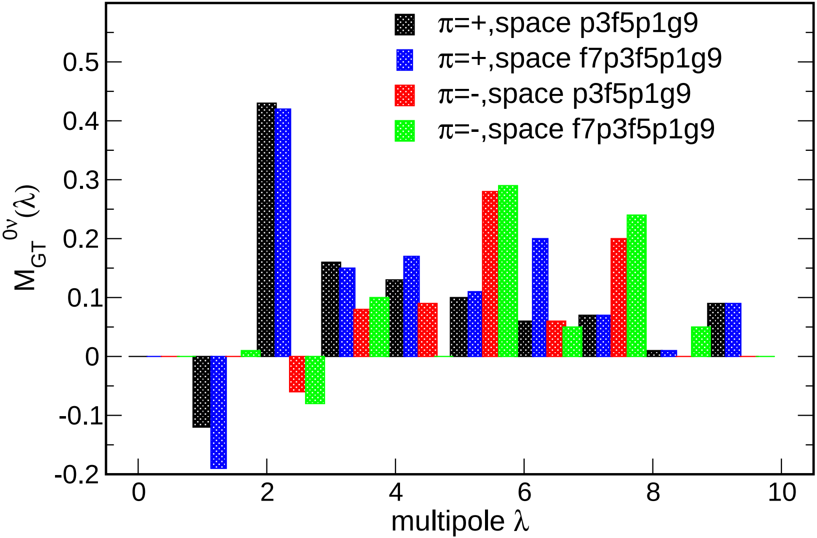
LSSM calculations in QRPA-like valence spaces: ^{82}Se

JJ-decomposition with and without $1f_{7/2}$ excitations



LSSM calculations in QRPA-like valence spaces: ^{82}Se

multipole decomposition with and without $1f_{7/2}$ excitations



CONCLUSIONS

- Large scale shell model calculations with high quality effective interactions are available or will be in the immediate future for all but one of the neutrinoless double beta emitters
- The theoretical spread of the values of the nuclear matrix elements entering in the lifetime calculations is greatly reduced if the ingredients of each calculation are examined critically and only those fulfilling a set of quality criteria are retained
- A concerted effort of benchmarking between LSSM and QRPA practitioners would be of utmost importance to increase the reliability and precision of the nuclear structure input for the double beta decay processes